Exercises

Exercise 1.1 Explain in one paragraph at least three levels of abstraction that are used by

(a) biologists studying the operation of cells.

(b) chemists studying the composition of matter.

Exercise 1.2 Explain in one paragraph how the techniques of hierarchy, modularity, and regularity may be used by

(a) automobile designers.

(b) businesses to manage their operations.

Exercise 1.3 Ben Bitdiddle is building a house. Explain how he can use the principles of hierarchy, modularity, and regularity to save time and money during construction.

Exercise 1.4 An analog voltage is in the range of 0–5 V. If it can be measured with an accuracy of ±50 mV, at most how many bits of information does it convey?

Exercise 1.5 A classroom has an old clock on the wall whose minute hand broke off.

(a) If you can read the hour hand to the nearest 15 minutes, how many bits of information does the clock convey about the time?

(b) If you know whether it is before or after noon, how many additional bits of information do you know about the time?

Exercise 1.6 The Babylonians developed the *sexagesimal* (base 60) number system about 4000 years ago. How many bits of information is conveyed with one sexagesimal digit? How do you write the number 4000_{10} in sexagesimal?

Exercise 1.7 How many different numbers can be represented with 16 bits?

Exercise 1.8 What is the largest unsigned 32-bit binary number?

Exercise 1.9 What is the largest 16-bit binary number that can be represented with

(a) unsigned numbers?

(b) two's complement numbers?

(c) sign/magnitude numbers?

Exercise 1.10 What is the largest 32-bit binary number that can be represented with

(a) unsigned numbers?

(b) two's complement numbers?

(c) sign/magnitude numbers?

Exercise 1.11 What is the smallest (most negative) 16-bit binary number that can be represented with

(a) unsigned numbers?

(b) two's complement numbers?

(c) sign/magnitude numbers?

Exercise 1.12 What is the smallest (most negative) 32-bit binary number that can be represented with

(a) unsigned numbers?

(b) two's complement numbers?

(c) sign/magnitude numbers?

Exercise 1.13 Convert the following unsigned binary numbers to decimal. Show your work.

(a) 1010_2

(b) 110110_2

(c) 11110000_2

(d) 0001000101001111_2

Exercise 1.14 Convert the following unsigned binary numbers to decimal. Show your work.

(a) 1110_2

(b) 100100_2

(c) 11010111_2

(d) 0111010101001000_2

Exercise 1.15 Repeat Exercise 1.13, but convert to hexadecimal.

Exercise 1.16 Repeat Exercise 1.14, but convert to hexadecimal.

Exercise 1.17 Convert the following hexadecimal numbers to decimal. Show your work.

(a) $A5_{16}$

(b) $3B_{16}$

(c) $FFFF_{16}$

(d) $D0000000_{16}$

Exercise 1.18 Convert the following hexadecimal numbers to decimal. Show your work.

(a) $4E_{16}$

(b) $7C_{16}$

(c) $ED3A_{16}$

(d) $403FB001_{16}$

Exercise 1.19 Repeat Exercise 1.17, but convert to unsigned binary.

Exercise 1.20 Repeat Exercise 1.18, but convert to unsigned binary.

Exercise 1.21 Convert the following two's complement binary numbers to decimal.

(a) 1010_2

(b) 110110_2

(c) 01110000_2

(d) 10011111_2

Exercise 1.22 Convert the following two's complement binary numbers to decimal.

(a) 1110_2

(b) 100011_2

(c) 01001110_2

(d) 10110101_2

Exercise 1.23 Repeat Exercise 1.21, assuming the binary numbers are in sign/magnitude form rather than two's complement representation.

Exercise 1.24 Repeat Exercise 1.22, assuming the binary numbers are in sign/magnitude form rather than two's complement representation.

Exercise 1.25 Convert the following decimal numbers to unsigned binary numbers.

(a) 42_{10}

(b) 63_{10}

(c) 229_{10}

(d) 845_{10}

Exercise 1.26 Convert the following decimal numbers to unsigned binary numbers.

(a) 14_{10}

(b) 52_{10}

(c) 339_{10}

(d) 711_{10}

Exercise 1.27 Repeat Exercise 1.25, but convert to hexadecimal.

Exercise 1.28 Repeat Exercise 1.26, but convert to hexadecimal.

Exercise 1.29 Convert the following decimal numbers to 8-bit two's complement numbers or indicate that the decimal number would overflow the range.

(a) 42_{10}

(b) -63_{10}

(c) 124_{10}

(d) -128_{10}

(e) 133_{10}

Exercise 1.30 Convert the following decimal numbers to 8-bit two's complement numbers or indicate that the decimal number would overflow the range.

(a) 24_{10}

(b) -59_{10}

(c) 128_{10}

(d) -150_{10}

(e) 127_{10}

Exercise 1.31 Repeat Exercise 1.29, but convert to 8-bit sign/magnitude numbers.

Exercise 1.32 Repeat Exercise 1.30, but convert to 8-bit sign/magnitude numbers.

Exercise 1.33 Convert the following 4-bit two's complement numbers to 8-bit two's complement numbers.

(a) 0101_2

(b) 1010_2

Exercise 1.34 Convert the following 4-bit two's complement numbers to 8-bit two's complement numbers.

(a) 0111_2

(b) 1001_2

Exercise 1.35 Repeat Exercise 1.33 if the numbers are unsigned rather than two's complement.

Exercise 1.36 Repeat Exercise 1.34 if the numbers are unsigned rather than two's complement.

Exercise 1.37 Base 8 is referred to as *octal*. Convert each of the numbers from Exercise 1.25 to octal.

Exercise 1.38 Base 8 is referred to as *octal*. Convert each of the numbers from Exercise 1.26 to octal.

Exercise 1.39 Convert each of the following octal numbers to binary, hexadecimal, and decimal.

(a) 42_8

(b) 63_8

(c) 255_8

(d) 3047_8

Exercise 1.40 Convert each of the following octal numbers to binary, hexadecimal, and decimal.

(a) 23_8

(b) 45_8

(c) 371_8

(d) 2560_8

Exercise 1.41 How many 5-bit two's complement numbers are greater than 0? How many are less than 0? How would your answers differ for sign/magnitude numbers?

Exercise 1.42 How many 7-bit two's complement numbers are greater than 0? How many are less than 0? How would your answers differ for sign/magnitude numbers?

Exercise 1.43 How many bytes are in a 32-bit word? How many nibbles are in the word?

Exercise 1.44 How many bytes are in a 64-bit word?

Exercise 1.45 A particular DSL modem operates at 768 kbits/sec. How many bytes can it receive in 1 minute?

Exercise 1.46 USB 3.0 can send data at 5 Gbits/sec. How many bytes can it send in 1 minute?

Exercise 1.47 Hard disk manufacturers use the term "megabyte" to mean 10^6 bytes and "gigabyte" to mean 10^9 bytes. How many real GBs of music can you store on a 50 GB hard disk?

Exercise 1.48 Estimate the value of 2^{31} without using a calculator.

Exercise 1.49 A memory on the Pentium II microprocessor is organized as a rectangular array of bits with 2^8 rows and 2^9 columns. Estimate how many bits it has without using a calculator.

Exercise 1.50 Draw a number line analogous to Figure 1.11 for 3-bit unsigned, two's complement, and sign/magnitude numbers.

Exercise 1.51 Draw a number line analogous to Figure 1.11 for 2-bit unsigned, two's complement, and sign/magnitude numbers.

Exercise 1.52 Perform the following additions of unsigned binary numbers. Indicate whether or not the sum overflows a 4-bit result.

(a) $1001_2 + 0100_2$

(b) $1101_2 + 1011^2$

Exercise 1.53 Perform the following additions of unsigned binary numbers. Indicate whether or not the sum overflows an 8-bit result.

(a) $10011001_2 + 01000100_2$

(b) $11010010_2 + 10110110_2$

Exercise 1.54 Repeat Exercise 1.52, assuming that the binary numbers are in two's complement form.

Exercise 1.55 Repeat Exercise 1.53, assuming that the binary numbers are in two's complement form.

Exercise 1.56 Convert the following decimal numbers to 6-bit two's complement binary numbers and add them. Indicate whether or not the sum overflows a 6-bit result.

(a) $16_{10} + 9_{10}$

(b) $27_{10} + 31_{10}$

(c) $-4_{10} + 19_{10}$

(d) $3_{10} + -32_{10}$

(e) $-16_{10} + -9_{10}$

(f) $-27_{10} + -31_{10}$

Exercise 1.57 Repeat Exercise 1.56 for the following numbers.

(a) $7_{10} + 13_{10}$

(b) $17_{10} + 25_{10}$

(c) $-26_{10} + 8_{10}$

(d) $31_{10} + -14_{10}$

(e) $-19_{10} + -22_{10}$

(f) $-2_{10} + -29_{10}$

Exercise 1.58 Perform the following additions of unsigned hexadecimal numbers. Indicate whether or not the sum overflows an 8-bit (two hex digit) result.

(a) $7_{16} + 9_{16}$

(b) $13_{16} + 28_{16}$

(c) $AB_{16} + 3E_{16}$

(d) $8F_{16} + AD_{16}$

Exercise 1.59 Perform the following additions of unsigned hexadecimal numbers. Indicate whether or not the sum overflows an 8-bit (two hex digit) result.

(a) $22_{16} + 8_{16}$

(b) $73_{16} + 2C_{16}$

(c) $7F_{16} + 7F_{16}$

(d) $C2_{16} + A4_{16}$

Exercise 1.60 Convert the following decimal numbers to 5-bit two's complement binary numbers and subtract them. Indicate whether or not the difference overflows a 5-bit result.

(a) $9_{10} - 7_{10}$

(b) $12_{10} - 15_{10}$

(c) $-6_{10} - 11_{10}$

(d) $4_{10} - -8_{10}$

Exercise 1.61 Convert the following decimal numbers to 6-bit two's complement binary numbers and subtract them. Indicate whether or not the difference overflows a 6-bit result.

(a) $18_{10} - 12_{10}$

(b) $30_{10} - 9_{10}$

(c) $-28_{10} - 3_{10}$

(d) $-16_{10} -21_{10}$

Exercise 1.62 In a *biased* N-bit binary number system with bias B, positive and negative numbers are represented as their value plus the bias B. For example, for 5-bit numbers with a bias of 15, the number 0 is represented as 01111, 1 as 10000, and so forth. Biased number systems are sometimes used in floating point mathematics, which will be discussed in Chapter 5. Consider a biased 8-bit binary number system with a bias of 127_{10}.

(a) What decimal value does the binary number 10000010_2 represent?

(b) What binary number represents the value 0?

(c) What is the representation and value of the most negative number?

(d) What is the representation and value of the most positive number?

Exercise 1.63 Draw a number line analogous to Figure 1.11 for 3-bit biased numbers with a bias of 3 (see Exercise 1.62 for a definition of biased numbers).

Exercise 1.64 In a *binary coded decimal* (BCD) system, 4 bits are used to represent a decimal digit from 0 to 9. For example, 37_{10} is written as 00110111_{BCD}.

(a) Write 289_{10} in BCD

(b) Convert 100101010001_{BCD} to decimal

(c) Convert 01101001_{BCD} to binary

(d) Explain why BCD might be a useful way to represent numbers

Exercise 1.65 Answer the following questions related to BCD systems (see Exercise 1.64 for the definition of BCD).

(a) Write 371_{10} in BCD

(b) Convert 000110000111_{BCD} to decimal

(c) Convert 10010101_{BCD} to binary

(d) Explain the disadvantages of BCD when compared to binary representations of numbers

Exercise 1.66 A flying saucer crashes in a Nebraska cornfield. The FBI investigates the wreckage and finds an engineering manual containing an equation in the Martian number system: $325 + 42 = 411$. If this equation is correct, how many fingers would you expect Martians to have?

Exercise 1.67 Ben Bitdiddle and Alyssa P. Hacker are having an argument. Ben says, "All integers greater than zero and exactly divisible by six have exactly two 1's in their binary representation." Alyssa disagrees. She says, "No, but all such numbers have an even number of 1's in their representation." Do you agree with Ben or Alyssa or both or neither? Explain.

Exercise 1.68 Ben Bitdiddle and Alyssa P. Hacker are having another argument. Ben says, "I can get the two's complement of a number by subtracting 1, then inverting all the bits of the result." Alyssa says, "No, I can do it by examining each bit of the number, starting with the least significant bit. When the first 1 is found, invert each subsequent bit." Do you agree with Ben or Alyssa or both or neither? Explain.

Exercise 1.69 Write a program in your favorite language (e.g., C, Java, Perl) to convert numbers from binary to decimal. The user should type in an unsigned binary number. The program should print the decimal equivalent.

Exercise 1.70 Repeat Exercise 1.69 but convert from an arbitrary base b_1 to another base b_2, as specified by the user. Support bases up to 16, using the letters of the alphabet for digits greater than 9. The user should enter b_1, b_2, and then the number to convert in base b_1. The program should print the equivalent number in base b_2.

Exercise 1.71 Draw the symbol, Boolean equation, and truth table for

(a) a three-input OR gate

(b) a three-input exclusive OR (XOR) gate

(c) a four-input XNOR gate

Exercise 1.72 Draw the symbol, Boolean equation, and truth table for

(a) a four-input OR gate

(b) a three-input XNOR gate

(c) a five-input NAND gate

Exercise 1.73 A *majority gate* produces a TRUE output if and only if more than half of its inputs are TRUE. Complete a truth table for the three-input majority gate shown in Figure 1.41.

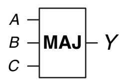

Figure 1.41 **Three-input majority gate**

Exercise 1.74 A three-input *AND-OR* (*AO*) *gate* shown in Figure 1.42 produces a TRUE output if both *A* and *B* are TRUE, or if *C* is TRUE. Complete a truth table for the gate.

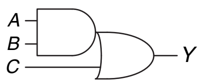

Figure 1.42 **Three-input AND-OR gate**

Exercise 1.75 A three-input *OR-AND-INVERT* (*OAI*) *gate* shown in Figure 1.43 produces a FALSE output if *C* is TRUE and *A* or *B* is TRUE. Otherwise it produces a TRUE output. Complete a truth table for the gate.

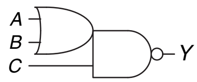

Figure 1.43 Three-input OR-AND-INVERT gate

Exercise 1.76 There are 16 different truth tables for Boolean functions of two variables. List each truth table. Give each one a short descriptive name (such as OR, NAND, and so on).

Exercise 1.77 How many different truth tables exist for Boolean functions of N variables?

Exercise 1.78 Is it possible to assign logic levels so that a device with the transfer characteristics shown in Figure 1.44 would serve as an inverter? If so, what are the input and output low and high levels (V_{IL}, V_{OL}, V_{IH}, and V_{OH}) and noise margins (NM_L and NM_H)? If not, explain why not.

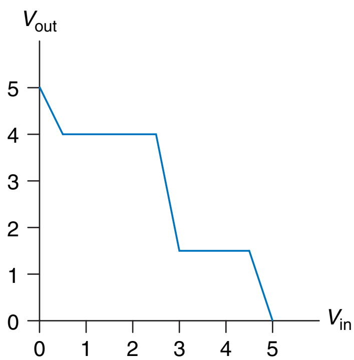

Figure 1.44 DC transfer characteristics

Exercise 1.79 Repeat Exercise 1.78 for the transfer characteristics shown in Figure 1.45.

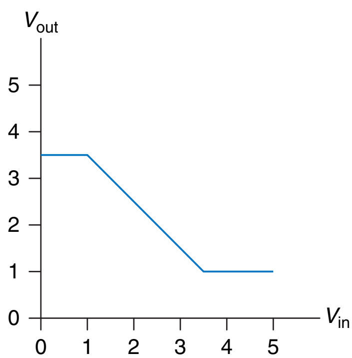

Figure 1.45 DC transfer characteristics

Exercise 1.80 Is it possible to assign logic levels so that a device with the transfer characteristics shown in Figure 1.46 would serve as a buffer? If so, what are the input and output low and high levels (V_{IL}, V_{OL}, V_{IH}, and V_{OH}) and noise margins (NM_L and NM_H)? If not, explain why not.

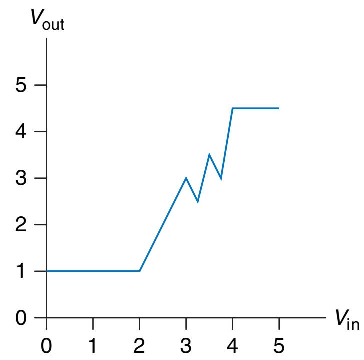

Figure 1.46 DC transfer characteristics

Exercise 1.81 Ben Bitdiddle has invented a circuit with the transfer characteristics shown in Figure 1.47 that he would like to use as a buffer. Will it work? Why or why not? He would like to advertise that it is compatible with LVCMOS and LVTTL logic. Can Ben's buffer correctly receive inputs from those logic families? Can its output properly drive those logic families? Explain.

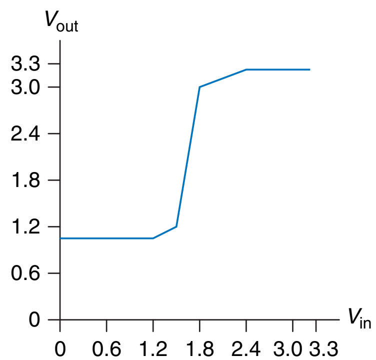

Figure 1.47 Ben's buffer DC transfer characteristics

Exercise 1.82 While walking down a dark alley, Ben Bitdiddle encounters a two-input gate with the transfer function shown in Figure 1.48. The inputs are A and B and the output is Y.

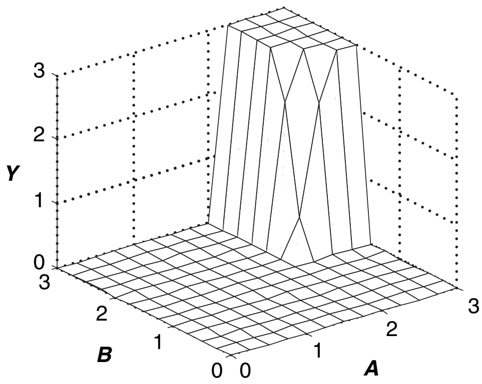

Figure 1.48 Two-input DC transfer characteristics

(a) What kind of logic gate did he find?

(b) What are the approximate high and low logic levels?

Exercise 1.83 Repeat Exercise 1.82 for Figure 1.49.

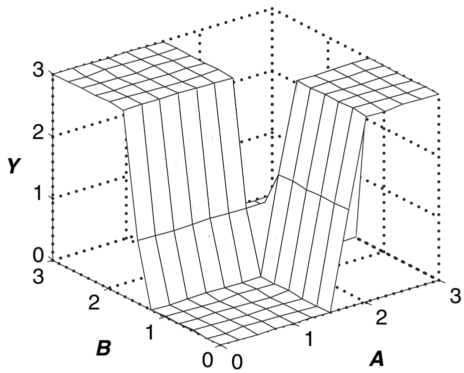

Figure 1.49 Two-input DC transfer characteristics

Exercise 1.84 Sketch a transistor-level circuit for the following CMOS gates. Use a minimum number of transistors.

(a) four-input NAND gate

(b) three-input OR-AND-INVERT gate (see Exercise 1.75)

(c) three-input AND-OR gate (see Exercise 1.74)

Exercise 1.85 Sketch a transistor-level circuit for the following CMOS gates. Use a minimum number of transistors.

(a) three-input NOR gate

(b) three-input AND gate

(c) two-input OR gate

Exercise 1.86 A *minority gate* produces a TRUE output if and only if fewer than half of its inputs are TRUE. Otherwise it produces a FALSE output. Sketch a transistor-level circuit for a three-input CMOS minority gate. Use a minimum number of transistors.

Exercise 1.87 Write a truth table for the function performed by the gate in Figure 1.50. The truth table should have two inputs, A and B. What is the name of this function?

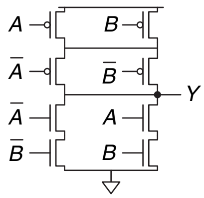

Figure 1.50 Mystery schematic

Exercise 1.88 Write a truth table for the function performed by the gate in Figure 1.51. The truth table should have three inputs, A, B, and C.

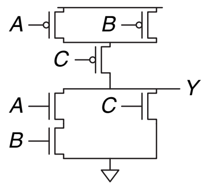

Figure 1.51 Mystery schematic

Exercise 1.89 Implement the following three-input gates using only pseudo-nMOS logic gates. Your gates receive three inputs, A, B, and C. Use a minimum number of transistors.

(a) three-input NOR gate

(b) three-input NAND gate

(c) three-input AND gate

Exercise 1.90 *Resistor-Transistor Logic* (*RTL*) uses nMOS transistors to pull the gate output LOW and a weak resistor to pull the output HIGH when none of the paths to ground are active. A NOT gate built using RTL is shown in Figure 1.52. Sketch a three-input RTL NOR gate. Use a minimum number of transistors.

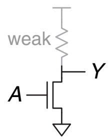

Figure 1.52 **RTL NOT gate**